Creating an Environment for Learning with Understanding: The Learning Principle

THE LEARNING PRINCIPLE IN NCTM’S Principles and Standards for School Mathematics (2000) states: “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” Learning with understanding is defined as “being able to apply procedures, concepts, and processes” (NCTM 2000, p. 20). This view of learning represents a departure from a view that emphasizes a student’s factual knowledge and ability to apply procedures. Although facts and procedures are important, they will not, in and of themselves, result in learning with understanding. Instead, factual understanding, procedural fluency, and conceptual understanding must coexist so that students reach learning with understanding. The extent to which a student can apply his or her learning to a new problem or situation is often an indicator of this understanding.

Learning and teaching are intricately connected, and enabling students to learn with understanding depends heavily on the opportunities that a teacher provides to his or her students. Moreover, as communication plays an important role in the formulation and transfer of understanding, classroom structures that encourage communication can enhance student learning. Principles and Standards underscores the importance of classroom culture and discourse. “In such settings, procedural fluency and conceptual understanding can be developed through problem solving, reasoning, and argumentation” (NCTM 2000, p. 21). This article uses a classroom example to illustrate what the Learning Principle might look like in practice.

Learning with Understanding in Algebra: A Classroom Vignette

opportunities to examine, extend, and generalize patterns enable students to learn algebra with understanding, which is a significant goal in the middle grades. Building conceptual algebraic understanding requires mathematical experiences

"Spotlight on the Principles" focuses on the six overarching principles for grades 6–8 found in NCTM’s Principles and Standards for School Mathematics (2000). The articles discuss how these principles relate to middle grades mathematics and suggest ways that teachers might incorporate them into their instruction.

The author wishes to thank the sixth graders and their teacher, Cathy Walker, at John Adams Middle School in Kanawha County, West Virginia, for opening their classroom to this project and for sharing the work that inspired this article.
and instruction that go beyond algorithmic procedures. Problems that entail building models to represent a situation and which ask students to connect a rule or expression to one’s model allow students to see the power of algebra as a problem-solving tool.

Learning with understanding can be promoted through the use of rich mathematical investigations that are set in real contexts. One such investigation was observed in a sixth-grade classroom that exemplifies best practice in instructional strategies and classroom discourse. The work of these students and their teacher reflected many aspects of the Learning Principle and inspired this article.

The sixth graders in Mrs. W.’s class were beginning the second lesson of an algebra unit called “Patterns in Numbers and Shapes” from MathScape: Seeing and Thinking Mathematically, a Standards-based middle-grades mathematics curriculum (Education Development Center 2005). After a carefully planned and concise introduction to the investigation, Mrs. W. asked students to work in small groups. Her introduction focused on explaining the task and checking for understanding of potentially confusing vocabulary, giving only enough information to get them started. Using fewer than five minutes of valuable class time for this introduction, Mrs. W.’s actions highlighted both her expectations that students take an active role in learning mathematics and her belief in their ability to do mathematics through carefully designed investigations administered in a particular classroom context (see fig. 1).

Most students began the task by using rods to build a structure and then counting the sides that were showing (see fig. 2). They could then complete the table in figure 3. Organizing the information in a table enabled them to identify an additive pattern: the stamps needed to be increased by 4 with each unit-length increase to the rod. Students could then complete the table to a length of 10 with relative ease. In addition, they could relate the answers in their table back to the model of the situation. Using the model, they could explain why the number of stamps needed to increase by 4 each time. Students used the model to look for the pattern and identified different approaches. They shared these strategies for finding a pattern:

Stephen: I noticed that every time you put on another cube, it adds 4 more onto the stamps.

Brittany: When you have two cubes connected,
there are only 5 sides shown, with 1 side of each cube hidden. So, I added those and I got 10. When I added one more cube, the middle cube is only going to have 4 sides showing because 2 of its sides are hidden. So, it would be $5 + 5 + 4$, or 14 stamps. When I got to adding the fourth cube, I looked at my table and saw the pattern of increasing by 4 with each cube.

This “adding-four method” soon became inefficient when students were asked to determine the number of paint stamps needed for a length of 66. Question 2 in figure 3, “Write a rule you could use to extend the pattern to any length rod,” required students to move beyond the iterative pattern of adding 4 to the previous term. Some students struggled to understand a method that could be used for any length rod; however, after working together, just about every student in the classroom seemed to know the rule and could relate it back to the model and the situation. One group’s exchange was as follows:

Mrs. W.: What if you wanted to know the stamps needed for a rod of length 25?
Stephen: You could just multiply by 4 because there are 4 sides.
Mrs. W.: Show me what you mean on the model.
Stephen: [Turning the model to show the 4 sides] Each cube has a side here, here, here, and here. So, you multiply by 4. Then add 2 for the end faces.
Mrs. W.: [Turning to Jennifer] Did you hear what he said?
Jennifer: Yes.
Mrs. W.: Can you explain it in your own words?
Jennifer: Yes. Every cube has at least 4 sides showing. You multiply the length of the rod by 4. Once you get this answer, you have to add 2 for the ends.

One student showed interest in applying her understanding of the pattern to another situation. She asked, “What if rather than building a straight, one-level rod, I put cubes on top? How would this change the pattern and the rule?” (See fig. 4.) Efforts to apply one’s understanding to novel problems indicate understanding that goes beyond factual knowledge or procedural fluency into conceptual understanding and, ultimately, to learning with understanding. This student is building understanding that she can use to solve new problems. Moreover, her curiosity in posing new problems indicates that she is taking control of her learning, a habit of mind that should serve her well in school and beyond.

The final part of the investigation asked students to apply their rules and models and work backward. If given the number of stamps needed to paint a rod, could they determine the length of the rod? This task challenged students to apply their conceptual understanding of the problem to solve for a different variable. Mrs. W. believes firmly that students can learn a lot from one another and will benefit from classroom opportunities to struggle through problems together, sharing their reasoning or explaining processes as they go. When interviewed after the lesson, Mrs. W. remarked, “A student who has just struggled through a problem and figured it out can often explain it to someone who is still struggling. . . . Working in groups can help them to obtain a deeper understanding of the math because teaching it to someone else helps them internalize the knowledge. It helps them reach mastery because their knowledge is not just what the answer is, but how to get that answer and why it works.”
the following exchange, the opportunity to verbalize his thinking allowed one student to refine his understanding and explain it to others:

*Mrs. W.*: Can you tell them how you would work backward from 42 stamps to a rod of length 10?

*Will.*: I would find out what times 4 is the closest to 42. That’s 11, right?

*Mrs. W.*: [To other group members] Is that what you think?

*Maria.*: [To another student] Could you repeat that? I’m not really sure.

*Will.*: Well. Eleven times. . . . No, wait. Ten times 4 equals 40. So, add the 2 ends and that would equal 42.

*Maria.*: Yeah.

*Will.*: So.

*Maria.*: I get it.

*Will.*: That’s how I got it.

Students’ success varied on this part of the lesson. For students to have learned with understanding, one would expect that they could model and use their rule to solve this problem. Although some were successful, many students had not attained this level of understanding. Mrs. W. was confident, however, that many more would be able to apply their rule to work backward in subsequent lessons in the unit.

**Promoting the Learning Principle in the Classroom**

During group debate and in the whole-class discussion that concluded the lesson, students often built on the ideas of one another. Classroom interactions such as these contribute to students’ learning with understanding. In addition, the classroom culture must value risk taking and protect the risk taker. Students who feel comfortable expressing themselves, even if making a mistake, will be more apt to engage in the activity and the conversation. Such opportunities to struggle through one’s ideas in a supportive environment can help students to learn with understanding. How a teacher poses questions and responds to answers impact how students feel about participating in the mathematics. How classmates contribute is equally as important to Mrs. W., who relies on students to answer the questions of other students. “I want students to see the different paths that others took toward the answer because that will help them on their next problem.”

According to the Learning Principle in *Principles and Standards*, “A major goal of school mathematics programs is to create autonomous learners” (NCTM 2000, p. 21). Mrs. W.’s high expectations and her belief in each student’s ability to learn mathematics contributed to the development of autonomous learners. Her disposition toward her students, coupled with the challenging problems that she provided, allowed them to gain self-confidence. With this confidence, they could then independently apply their learning successfully in new situations.

Although the Learning Principle is centered on what students should be able to do, it is clearly impacted by what the teacher does. Learning with understanding depends on both the opportunities offered to students and the environment in which students work. The classroom environment should value the contributions of students, hear all voices, maintain high expectations for all students, and foster a sense of community among learners. The classroom should be a place where students are challenged, where they feel safe to take intellectual risks, and where students are responsible for their own learning. An eye toward these conditions can foster learning with understanding.

The following are questions to think about when examining a classroom environment:

- Who does most of the talking? Most of the work?
- Do students have opportunities to share their thinking?
- Are students expected to share their work in a public way?
- How do students feel when their answers are wrong?
- Do students have opportunities to see more than one solution to a problem?
- Who participates in class discussion?
- How does the teacher frame questions to students? How does he or she respond to students?

The following are questions to think about when evaluating a task for mathematics class. Does the task—

- encourage students to explain their reasoning or approach and defend their answer?
• offer flexibility in the problem-solving approach that is used?
• have multiple acceptable answers or solution methods, with varied levels of sophistication?
• provide access to students with a range of skills and abilities?
• allow different points of entry or exit?
• offer challenge and rigor?
• address important mathematical goals?

Summary

IN THE LEARNING PRINCIPLE, PRINCIPLES AND Standards contends that students learn more and learn better when they can take control of their learning by defining their goals and monitoring their progress. In posing reflection questions to students at the end of a discussion, Mrs. W. moves students in this direction. Before dismissing the students, she asked her class, “OK, I’d like three things that you learned during your work here today.” One student replied, “I learned that you can find out if there are patterns in what you do, and then if you find a pattern it will make it a lot easier to solve.”

It appears as though some learning with understanding has taken place, and algebra, rather than being a confusing, difficult mathematics subject, may actually be a very useful tool that is accessible to all. Instilling in students this belief was one of Mrs. W.’s goals and one that should serve students well as they work toward learning algebra with understanding.

References
